

$$E = \iiint_V \rho e \, dv$$

$$\frac{DE}{Dt} = \dot{Q} - \dot{W}$$

$$\frac{D}{Dt} \iiint_V \rho e \, dv = \iiint_V \frac{\partial \rho e}{\partial t} \, dv + \oint_S \rho e \vec{v} \cdot \vec{n} \, ds = \iiint_V q''' \, dv - \oint_S \vec{q}'' \cdot \vec{n} \, dv - \dot{W} \oint_S \vec{v} \cdot \vec{n} \, dv$$

e' desprezado

$$\frac{\partial \rho e}{\partial t} + \vec{\nabla} \cdot (\rho e \vec{v}) = q''' - \vec{\nabla} \cdot \vec{q}'' \quad q'' = -k \nabla T$$

$$\rho c_p \left(\frac{\partial T}{\partial t} + (\vec{v} \cdot \vec{\nabla}) T \right) = \nabla \cdot (k \nabla T) + q'''$$

$$\rho c_p \frac{\partial T}{\partial t} = \vec{\nabla} \cdot (k \nabla T) + q''' \quad (8)$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \vec{v} = -\frac{1}{\rho} \nabla p + \frac{\mu}{\rho} \nabla^2 \vec{v} \quad (9)$$

$$\vec{\nabla} \cdot \vec{v} = 0 \quad (9)$$

Estudando a equação (8)

Caso estacionário: $0 = \nabla \cdot (k \nabla T) + q''' \quad k \text{ constante}$
 $0 = k \nabla^2 T + q'''$

$$0 = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + q''' \quad \text{coord. cartesianas}$$

$$0 = k \left(\frac{\partial^2 T}{\partial z^2} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{r^2 \partial \theta^2} \right) + q''' \quad \text{coord. cilíndricas}$$

$$0 = k \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial^2 T}{\partial \varphi^2} \right) + q''' \quad \text{coord. esféricas}$$